# Problem 6 : Separable Functions

$$f(x_1, x_2, \ldots, x_n) = \prod_{j=1}^{k} (\alpha_{j,1} x_{i_{j,1}} + \alpha_{j,2} x_{i_{j,2}} - \min(\alpha_{j,1}, 0) - \min(\alpha_{j,2}, 0))$$

$$\alpha_{j,1}, \alpha_{j,2} = -1, 1$$

- we are only interested in the value of $f$ at $x_1, \ldots, x_n = 0, 1$.
- Note that each term evaluates either to 0 or 1, and when $\alpha = -1$, you actually have 1-x in the corresponding term.
- *e.g.* $(x_1 + x_2)(x_3 + x_4)$ becomes $(x_1 \vee x_2) \wedge (x_3 \vee x_4)$
- each term can be split into two implications: e.g. $x_1 \vee x_2$ makes $(\sim x_1 \rightarrow x_2) \wedge (\sim x_2 \rightarrow x_1)$
- these implications can make chains, e.g. $x_1 \rightarrow x_2 \rightarrow \sim x_3$ and if such a chain contains a loop $x \rightarrow \sim x$ and $\sim x \rightarrow x$ we have a contradiction, hence $f = 0$ for all inputs.

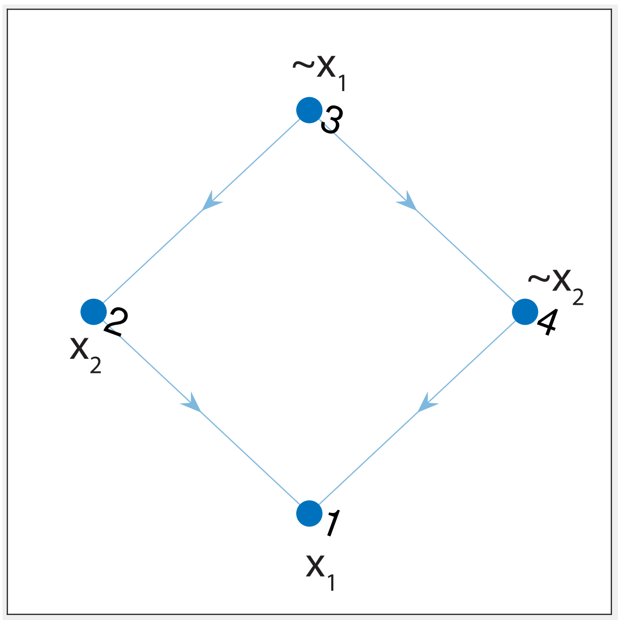
2
2
1 1 1 2
1 -1 1 2

$f = (x_1 + x_2)(x_1 + 1 - x_2)$

$(x_1 \text{ or } x_2)$ and $(x_1 \text{or} \sim x_2)$

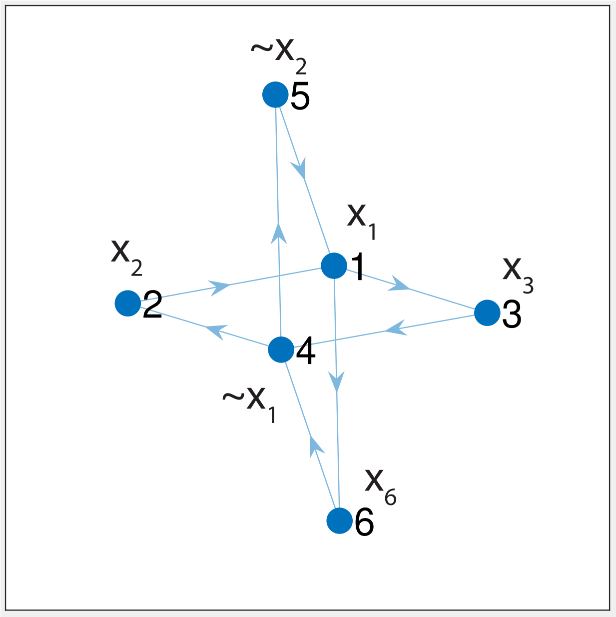
$\sim x_1 \rightarrow x_2$ $\qquad$ $\sim x_1 \rightarrow \sim x_2$

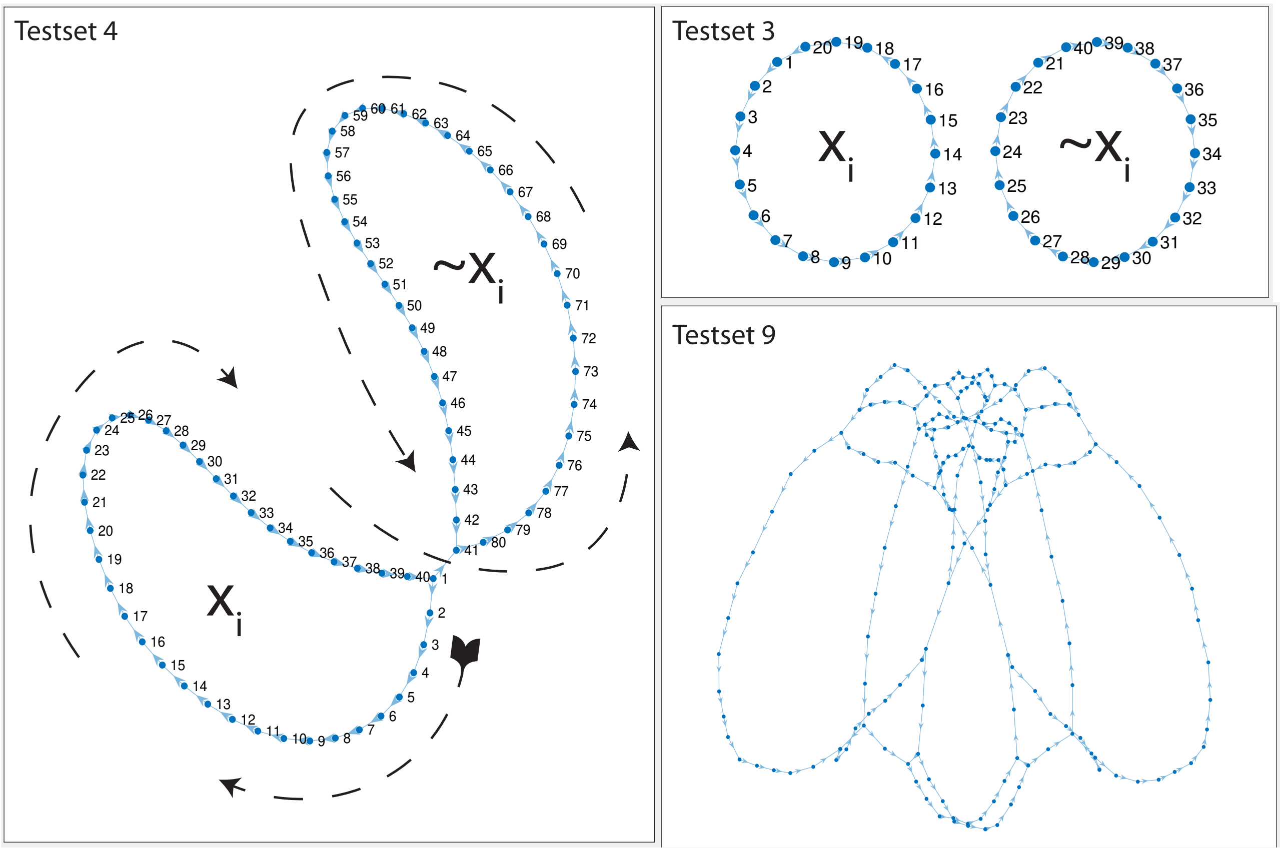
$\sim x_2 \rightarrow x_1$ $\qquad$ $x_2 \rightarrow x_1$




*Contra example* $x_1 = 1,\ x_2 = 1,\ \text{f} = 1$

3
4
1 1 1 2
1 -1 1 2
-1 1 1 3
-1 -1 1 3



Testset 5: